(i) $b=a, a=0$;

(ii) $b=a, a\neq 0$;

(iii) $b\neq a, a=0$;

(iv) $b\neq a, a\neq 0$.

Special case (i) is historically the oldest used in practice and obviously most restricting the parameters' freedom of variation. Notably, it is still the one routinely considered in publications like "System of National Accounts" (1993) and "Handbook of Price and Volume Measures" (2001). Because of its relevance, we also focus attention on that case.

In F_2 we first multiply K_t and K_b of the quantity index in (2) by $(\sum q_{ha}p_{ht}/\sum q_{ha}p_{ht})=1$ and $\sum q_{ha}p_{hb}/\sum q_{ha}p_{hb})=1$, respectively:

$$\frac{K_t\sum q_{ha}p_{ht}}{\sum q_{ha}p_{ht}}/\frac{K_b\sum q_{ha}p_{hb}}{\sum q_{ha}p_{hb}} \tag{4}$$

and next, realizing that $K_t q_{ha}=q_{ht}$ and $K_b q_{ha}=q_{hb}$ by definition, we are led to an algebraically equivalent form of (2):

$$\frac{K_t(\sum q_{ha}p_{ht})/K_t}{K_b(\sum q_{ha}p_{hb})/K_b}\times\frac{\dfrac{\sum q_{ht}p_{ht}}{\sum q_{ha}p_{ht}}}{\dfrac{\sum q_{hb}p_{hb}}{\sum q_{ha}p_{hb}}}=\frac{K_t\sum q_{ha}p_{ht}}{K_b\sum q_{ha}p_{hb}} \tag{5}$$

and, after substituting $b=a=0$ in (5), the needed special form of F_2 follows:

$$\frac{\sum q_{h0}p_{ht}}{\sum q_{h0}p_{h0}}\times\frac{\dfrac{\sum q_{ht}p_{ht}}{\sum q_{h0}p_{ht}}}{1(c.u.)}=\frac{\sum q_{ht}p_{ht}}{\sum q_{h0}p_{h0}} \tag{6}$$

where $(c.u.)$ means compound unit.

Next, turning to F_3, substituting and simplifying in (3), the special form follows: