

Therefore the density function can be expressed as a mixture of densities

$$f(x_1, x_2, d_1, d_2) = (f(x_1, x_2)(1 - G_1(x_1))(1 - G_2(x_2)))^{d_1 d_2} \times \\ \times \left(\frac{\partial}{\partial x_1}(1 - F(x_1, x_2))(1 - G_1(x_1))g_2(x_2)\right)^{d_1(1-d_2)} \times \\ \times \left(\frac{\partial}{\partial x_2}(1 - F(x_1, x_2))(1 - G_2(x_1))g_1(x_2)\right)^{(1-d_1)d_2} (S(x_1, x_2))^{(1-d_1)(1-d_2)}.$$

As the censoring times $Y_{ij}, i=1, \dots, n; j=1, 2$ are not informative, in the likelihood function only the terms which give information will be included. So, the likelihood curve for a given observation (x_p, x_p, d_p, d_p) will be

$$l(x_1, x_2, d_1, d_2) = f(x_1, x_2)^{d_1 d_2} \frac{\partial}{\partial x_1}(1 - F(x_1, x_2))^{d_1(1-d_2)} \times \\ \times \frac{\partial}{\partial x_2}(1 - F(x_1, x_2))S(x_1, x_2)^{(1-d_1)(1-d_2)}.$$

The most common type of truncation is left truncation. We can assume that truncation is not random. Left truncation arises when individuals come under observation only some known time after the natural origin of the event under study. So, if the individual fails before the study begins, that individual will not be recorded.

If there is left truncation we can use the same density function, but only if the individuals survive the truncation moment T^* .

$$P(T_1 > t_1, T_2 > t_2, d_1, d_2 \mid T_1 > T^*, T_2 > T^*) = P(T_1 > t_1, T_2 > t_2, d_1, d_2, T_1 > T^*, T_2 > T^*) \\ = \frac{f(x_1, x_2, d_1, d_2)}{S(y_1, y_2)},$$

where $y_i = T^* - b_i$. Here with b_i is denoted the time of birth. Therefore the likelihood curve for such a model can be expressed as

$$l(T_1, T_2, d_1, d_2 \mid T_1 > T^*, T_2 > T^*) = \frac{l(T_1, T_2, d_1, d_2)}{S(y_1, y_2)}.$$

In general, the loglikelihood function in this case can be calculated as follows:

$$L((T_{i1}, T_{i2}, \Delta_{i1}, \Delta_{i2}), i = 1, \dots, n) = \sum_{i=1}^n \log l(T_{i1}, T_{i2}, \Delta_{i1}, \Delta_{i2} \mid T_1 > T^*, T_2 > T^*).$$