Further, let the times of death of two related individuals are T_1, T_2, and the frailties of these two individuals are Z_1 , Z_2. According to Weinke (2001), we can decompose them in this way:

$$Z_1 = \frac{\lambda_0}{\lambda_1} Z_c + Z_{d1}$$

$$Z_2 = \frac{\lambda_0}{\lambda_2} Z_c + Z_{d2},$$

where $Z_c \in \Gamma(k_0, \lambda_0)$ represents the common genetic information and environment and $Z_{d1} \in \Gamma(k_1, \lambda_1)$, $Z_{d2} \in \Gamma(k_2, \lambda_2)$ represent the difference between the in dividuals in both their genetic information and environmental parameters. Hence $Z_1 = \Gamma(k_0 + k_1, \lambda_1)$ and $Z_1 = \Gamma(k_0 + k_2, \lambda_2)$. Therefore

$$EZ_1 = EZ_2 = 1,\ DZ_1 = \tfrac{1}{\lambda_1} = \sigma_1^2,\ DZ_1 = \tfrac{1}{\lambda_1} = \sigma_1^2.$$

Even more

$$EZ_1Z_2 = \frac{k_0}{(k_0 + k_1)(k_0 + k_2)} + 1$$

and

$$cov(Z_1, Z_2) = \frac{k_0}{(k_0 + k_1)(k_0 + k_2)}.$$

Therefore, for the correlation coefficient between the two frailty variables we have

$$\rho = \frac{cov(Z_1, Z_2)}{\sqrt{DZ_1 DZ_2}} = \frac{k_0}{\sqrt{(k_0 + k_1)(k_0 + k_2)}}.$$

Consequently, as $k_0 + k_i = \lambda_i = \frac{1}{\sigma_i^2}, i = 1, 2$ we have $k_0 = \frac{\rho}{\sigma_1 \sigma_2}$ and $k_i = \frac{1 - \frac{\sigma_i}{\sigma_j}\rho}{\sigma_i^2}, i, j = 1, 2, i \neq j.$

Then the unconditional survival function for this model is

$$S(x_1, x_2) = (1 + \sigma_1^2 H(x_1) + \sigma_2^2 H(x_2))^{-\frac{\rho}{\sigma_1\sigma_2}} (1 + \sigma_1^2 H(x_1))^{-\frac{1-\frac{\sigma_1}{\sigma_2}\rho}{\sigma_1^2}} (1 + \sigma_2^2 H(x_2))^{-\frac{1-\frac{\sigma_2}{\sigma_1}\rho}{\sigma_2^2}},$$

where $\rho = \frac{k_0}{\lambda_1 \lambda_2}$ nd $H(x) = \int_0^x \mu_0(u)du$ is the cumulative hazard function.

It is assumed that the baseline hazard function follows the Gompertz law $\mu_0(x) = ae^{bx}$, but it is not difficult to extend the model with the hazard function $\mu_0(x) = ae^{bx} + c$ The parameters of the model can be estimated by using the maximum likelihood estimate (MLE) for a density function

$$f(x_1, x_2) = \frac{\partial^2}{\partial x_1 \partial x_2} S(x_1, x_2).$$

Let us remark that if $Z_i = 0, i = 1, 2$ ve have the Shared Frailty Model.