Another approach is to include a random variable that represents the difference between the individuals in the model. This variable Z is called frailty. By using different distributions for Z different models can be obtained. Let us suppose that the hazard function of an individual can be presented as follows:

$$\mu(x, Z) = Z\mu_0(x),$$

where $\mu_0(x)$ is the underlying hazard. Let us note with $S(x|Z)$ the survival function of the individual with frailty Z. Therefore we have

$$S(x \mid Z) = e^{-\int_0^x \mu(s,Z)ds} = e^{-Z\int_0^x \mu_0(s)ds} = e^{-ZH_0(x)},$$

where

$$H_0(x) = \int_0^x \mu_0(s)ds$$

is the cumulative hazard function. So, the observed hazard function in the population is

$$S(x) = ES(x \mid Z) = Ee^{-ZH_0(x)} = L(H_0(x)),$$

where $L(H_0(x))$ is the Laplasian transformation. For the probability density function of the lifespan we have

$$f(x) = -\mu_0(x)L'(H_0(x)).$$

By definition we have

$$\mu(x, Z) = \frac{f(x \mid Z)}{S(x \mid Z)} = Z\mu_0(x),$$

therefore

$$f(x \mid Z) = Z\mu_0(x)S(x \mid Z),$$

where

$$f(x) = \mu_0(x)\int_0^\infty ZS(x \mid Z)f_z(z)dz.$$

For the mortality in the population we have

$$\mu(x) = \frac{\mu_0(x)\int_0^\infty ZS(x \mid Z)f_z(z)dz}{S(x)}.$$

As $f(z, X > x) = f_Z(z)S(x \mid Z)$, then

$$f(z \mid X > x) = \frac{S(x \mid Z)f_z(z)}{S(x)}.$$

So for the observed hazard function in the population we have

$$\mu(x) = \mu_0(x)\int_0^\infty zf(z \mid X > x)dz.$$