Here the Bivariate Survival analysis and Shared Frailty model are presented, Correlated Frailty is considered as well as the bivariate censoring and truncation problem are derived.

2. BIVARIATE SURVIVAL FUNCTION AND THE FRAILTY MODELS

Let T_1, T_2 be dependent lifespans and $S_i(x_i) = P(T_i > x_i), i = 1, 2$ be absolutely continuous univariate survival functions and let $S(x_1, x_2) = P(T_1 > x_1, T_2 > x_2)$ be a bivariate survival functions.

The function that is most often usedu demographics and survival analysis is the hazard function

$$\mu_i(x) = \lim_{\Delta \to 0} \frac{P(x_i < X_i < x_i + \Delta \mid X_i > x_i)}{\Delta} = \frac{f(x_i)}{1 - F(x_i)}, i = 1, 2,$$

where $f(x_i)$ and $F(x_i)$ are the probability density function and the cumulative density function of the lifespan $T_i, i = 1, 2$.

There are two conditional hazards related to $S(x_1, x_2)$. The first, $\bar{\mu}_i(x_i, x_j)$, represents the hazard of failure for T_i given $T_j > x_j, i = 1, 2, i \neq j$, and is defined as

$$\bar{\mu}_i(x_i, x_j) = \lim_{\Delta \to 0} \frac{1}{\Delta} P(x_i \leq T_i < x_i + \Delta \mid T_i > x_i, T_j > x_j) =$$

$$= -\frac{\partial}{\partial x_i} lnS(x_i, x_j).$$

The second one, $\tilde{\mu}_i(x_i; x_j)$, represents the hazard of failure at a moment T_i given $T_j = x_j$:

$$\tilde{\mu}_i(x_i, x_j) = \lim_{\Delta \to 0} \frac{1}{\Delta} P(x_i \leq T_i < x_i + \Delta \mid T_i > x_i, T_j = x_j) =$$

$$= -\frac{\partial}{\partial x_i} ln(\frac{\partial}{\partial x_j} S(x_i, x_j)).$$

One way of deriving a bivariate survival model is based on the introduction of a relation between these hazards (Yashin and Iachine, 1999). For example, the condition

$$\bar{\mu}_i(x_i, x_j) = (1 - \theta)\tilde{\mu}_i(x_i, x_j)$$

defines a bivariate survival function

$$S(x_1, x_2) = (S_1(x_1)^{-\theta} + S_2(x_2)^{-\theta} - 1)^{-\frac{1}{\theta}}.$$